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B_c Decays and Lifetime in QCD Sum Rules

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Abstract

In the framework of three-point QCD sum rules, the form factors for the semileptonic decays of $B_c^+ \to B_s(B_s^*)l^+\nu_l$ are calculated with account for the Coulomb-like α_s/v-corrections in the heavy quarkonium. The generalized relations due to the spin symmetry of HQET/NRQCD for the form factors are derived at the recoil momentum close to zero. The nonleptonic decays are studied using the assumption on the factorization. The B_c meson lifetime is estimated by summing up the dominating exclusive modes in the $c \to s$ transition combining the current calculations with the previous analysis of $b \to c$ decays in the sum rules of QCD and NRQCD.

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1 Introduction

For better understanding and precise measuring the weak-action properties of heavy quarks, governed by the QCD forces, we need as wide as possible collection of snapshots with hadrons, containing the heavy quarks. Then we can provide the study of heavy quarks dynamics by testing the various conditions, determining the forming of bound states as well as the entering of strong interactions into the weak processes. So, a new lab for such investigations is a doubly heavy long-lived quarkonium B_c recently observed by the CDF Collaboration [1] for the first time.

This meson is similar to the charmonium and bottomonium in the spectroscopy, since it is composed by two nonrelativistic heavy quarks, so that the NRQCD approach [2] is well justified to the system. The modern predictions for the mass spectra of $\bar{b}c$ levels were obtained in refs. [3] in the framework of potential models and lattice simulations. The arrangement of excitations is close to what was observed in the charmonium and bottomonium. However, the feature of B_c-mesons is an absence of annihilation into light quarks, gluons and leptons due to QCD and QED, that implies the higher excitations decay into the low lying levels and ground state due to the emission of photons and pion pairs. The measured value of B_c mass yet has a large uncertainty

$$M_{B_c} = 6.40 \pm 0.39 \pm 0.13 \text{ GeV},$$

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in agreement with the theoretical expectations.

The production mechanism for the B_c-meson was studied in refs. [4]. The most simple picture takes place for the production in the e^+e^--annihilation, where the universal perturbative fragmentation functions can be analytically calculated for the S-, P- and D-wave levels in the framework of factorization for the hard production of quarks and their soft binding into the hadron, which can be reliably described in the potential models. In hadron collisions, the fragmentation regime takes place at the transverse momenta $p_T \gg m_{B_c}$, and at $p_T \sim m_{B_c}$ the subleading terms in $1/p_T$ or higher twists have to be taken into account. This can be calculated in the framework of factorization approach by a careful evaluation of complete set of diagrams in the given α_s-order, $O(\alpha_s^4)$. The non-fragmentational contributions dominate at $p_T \sim m_{B_c}$ [4].

The measured B_c lifetime

$$\tau[B_c] = 0.46^{+0.18}_{-0.16} \pm 0.03 \text{ ps},$$

agrees with the estimates obtained in the framework of both the OPE combined with the NRQCD evaluation of hadronic matrix elements [5, 6, 7] and potential quark models, where one has to sum up the dominating exclusive modes to calculate the total B_c width [8, 9]

$$\tau_{\text{OPE,PM}}[B_c] = 0.55 \pm 0.15 \text{ ps}.$$

The accurate measurement of B_c lifetime could allow one to distinguish various parameter dependencies such as the optimal heavy quark masses, which basically determine the theoretical uncertainties in OPE.

At present, the calculations of B_c decays in the framework of QCD sum rules were performed in [10, 11, 12, 13]. The authors of [10, 11] got the results, where the form factors are about 3 times less than the values expected in the potential quark models, and the semileptonic and hadronic widths of B_c are one order of magnitude less than those in OPE. The reason for such the disagreement was pointed out in [12] and studied in [13]: in the QCD sum rules for the heavy quarkonia the Coulomb-like corrections are significant, since they correspond to summing up the ladder diagrams, where α_s/v is not a small parameter, as the heavy quarks move nonrelativistically, $v \ll 1$. The Coulomb rescaling of quark-quarkonium vertex enhances the estimates of form factors in the QCD sum rules for the $B_c^+ \to \psi(\eta_c) l^+ \nu$ decays, where the initial and recoil mesons are both the heavy quarkonia. In the framework of NRQCD at the recoil momentum close to zero one derives the spin symmetry relations for the form factors of semileptonic B_c decays [14, 13]. In the strict limit of $v_1 = v_2$, where $v_{1,2}$ denote the four-velocities of initial and recoil mesons, respectively, the authors of [14] found a single relation between the form factors [4]. In [13] the soft limit $v_1 \cdot v_2 \to 1$ at $v_1 \neq v_2$ was considered, and the generalized spin symmetry relations were obtained for the $B_c \to \psi(\eta_c)$ transitions: four equations, including that of [14]. Moreover, the gluon condensate term was calculated in both QCD and NRQCD, so that it enforced a convergency of the method.

In the present paper we calculate the B_c decays due to the $c \to s$ weak transition in the framework of QCD sum rules, taking into account the Coulomb-like α_s/v-corrections for the heavy quarkonium in the initial state. In the semileptonic decays the hadronic final state is saturated by the pseudoscalar B_s and vector B_s^* mesons, so that we need the values of their leptonic constants entering the sum rules and determining the normalization of form factors. For this purpose, we reanalyze the two-point sum rules for the B mesons to take into account the product of quark and gluon condensates in addition to the previous consideration of terms with the quark and mixed condensates. We demonstrate the

[4] In refs.[15, 16] the relations were studied in the framework of potential models.

significant role of the product term for the convergency of method and reevaluate the constants f_B as well as f_{B_s}. Taking into account the dependence on the threshold energy E_c of hadronic continuum in the $\bar{b}s$ system in both the value of f_{B_s} extracted from the two-point sum rules and the form factors in the three-point sum rules, we observe the stability of form factors versus E_c, which indicates the convergency of sum rules.

The spin symmetries of leading terms in the lagrangians of HQET [17] for the singly heavy hadrons (here $B_s^{(*)}$) and NRQCD [2] for the doubly heavy mesons (here B_c) result in the relations between the form factors of semileptonic $B_c \to B_s^{(*)}$ decays. We derive two generalized relations in the soft limit $v_1 \cdot v_2 \to 1$: one equation in addition to what was found previously in ref.[14]. The relations are in a good agreement with the sum rules calculations up to the accuracy better than 10%, that shows a low contribution of next-to-leading $1/m_Q$-terms.

We perform the numerical estimates of semileptonic B_c widths and use the factorization approach [18] to evaluate the hadronic modes. Summing up the dominating exclusive modes, we calculate the lifetime of B_c, which agree with the experimental data and the predictions of OPE and quark models. We discuss the preferable prescription for the normalization point of nonleptonic weak lagrangian for the charmed quark and present our optimal estimate of total B_c width. We stress that in the QCD sum rules to the given order in α_s, the uncertainty in the values of heavy quark masses is much less than in OPE. This fact leads to a more definite prediction on the B_c lifetime.

The paper is organized as follows. Section 2 is devoted to the general formulation of three-point sum rules for the B_c decays with account of Coulomb-like corrections. The analysis of two-point sum rules for the leptonic constant of singly heavy meson with the introduction of term allowing for the product of quark and gluon condensates is presented in Section 3, where we also show the convergency of three-point sum rules with respect to a dependence on the threshold energy of continuum in the heavy-light system. We estimate the form factors of semileptonic $B_c \to B_s^{(*)}$ decays. The relations between the form factors of semileptonic decays as follows from the spin symmetry of HQET and NRQCD are derived in Section 4 in the soft limit of zero recoil. Section 5 contains the description how the nonleptonic decays modes are calculated and the B_c lifetime is evaluated. We discuss the optimal estimation of lifetimes for the heavy hadrons in Section 6. In Conclusion we summarize the results.

2 Three-point sum rules for the B_c meson.

In this paper the approach of three-point QCD sum rules [19] is used to study the form factors of semileptonic and nonleptonic decay rates for the c $\to$ s transition in decays of B_c meson. From the two-point sum rules we extract the values for the leptonic constants of mesons in the initial and final states. In our consideration we use the following notations:

$$\langle 0|\bar{q}_1 i\gamma_5 q_2|P(p)\rangle = \frac{f_P M_P^2}{m_1 + m_2}, \tag{1}$$

and

$$\langle 0|\bar{q}_1 i\gamma_\mu q_2|V(p,\epsilon)\rangle = i\epsilon_\mu M_V f_V, \tag{2}$$

where P and V represent the scalar and vector mesons, and m_1, m_2 are the quark masses.

The hadronic matrix elements for the semileptonic $c \to s$ transition in the B_c decays can be written

down as follows:

$$\begin{aligned}
\langle B_s(p_2)|V_\mu|B_c(p_1)\rangle &= f_+(p_1+p_2)_\mu + f_- q_\mu, && (3)\\
\frac{1}{i}\langle B_s^*(p_2)|V_\mu|B_c(p_1)\rangle &= iF_V\epsilon_{\mu\nu\alpha\beta}\epsilon^{*\nu}(p_1+p_2)^\alpha q^\beta, && (4)\\
\frac{1}{i}\langle B_s^*(p_2)|A_\mu|B_c(p_1)\rangle &= F_0^A\epsilon_\mu^* + F_+^A(\epsilon^*\cdot p_1)(p_1+p_2)_\mu + F_-^A(\epsilon^*\cdot p_1)q_\mu, && (5)
\end{aligned}$$

where $q_\mu=(p_1-p_2)_\mu$ and $\epsilon^\mu=\epsilon^\mu(p_2)$ is the polarization vector of B_s^* meson. V_μ and A_μ are the flavour changing vector and axial electroweak currents. The form factors $f_\pm, F_V, F_0^A$ and $F_\pm^A$ are functions of q^2 only. It should be noted that since the leptonic current $l_\mu=\bar{l}\gamma_\mu(1+\gamma_5)\nu_l$ is transversal in the limit of massless leptons, the probabilities of semileptonic decays are independent of f_- and F_-^A (the $\tau^+\nu_\tau$ mode is forbidden by the energy conservation). Following the standard procedure for the evaluation of form factors in the framework of QCD sum rules [20], we consider the three-point functions

$$\begin{aligned}
\Pi_\mu(p_1,p_2,q^2) &= i^2\int dxdy e^{i(p_2\cdot x-p_1\cdot y)}\cdot\\
&\quad \langle 0|T\{\bar{q}_1(x)\gamma_5 q_2(x), V_\mu(0), \bar{b}(y)\gamma_5 c(y)\}|0\rangle, && (6)\\
\Pi_{\mu\nu}^{V,A}(p_1,p_2,q^2) &= i^2\int dxdy e^{i(p_2\cdot x-p_1\cdot y)}\cdot\\
&\quad \langle 0|T\{\bar{q}_1(x)\gamma_\mu q_2(x), J_\nu^{V,A}(0), \bar{b}(y)\gamma_5 c(y)\}|0\rangle, && (7)
\end{aligned}$$

where $\bar{q}_1(x)\gamma_5 q_2(x)$ and $\bar{q}_1(x)\gamma_\nu q_2(x)$ denote interpolating currents for B_s and B_s^*, correspondingly. $J_\mu^{V,A}$ are the currents V_μ and A_μ of relevance to the various cases.

The Lorentz structures in the correlators can be written down as

$$\begin{aligned}
\Pi_\mu &= \Pi_+(p_1+p_2)_\mu + \Pi_- q_\mu, && (8)\\
\Pi_{\mu\nu}^V &= i\Pi_V\epsilon_{\mu\nu\alpha\beta}p_2^\alpha p_1^\beta, && (9)\\
\Pi_{\mu\nu}^A &= \Pi_0^A g_{\mu\nu} + \Pi_1^A p_{2,\mu}p_{1,\nu} + \Pi_2^A p_{1,\mu}p_{1,\nu} + \Pi_3^A p_{2,\mu}p_{2,\nu} + \Pi_4^A p_{1,\mu}p_{2,\nu}. && (10)
\end{aligned}$$

The form factors $f_\pm$, F_V, F_0^A and $F_\pm^A$ are determined from the amplitudes $\Pi_\pm$, Π_V, Π_0^A and $\Pi_\pm^A = \frac{1}{2}(\Pi_1^A \pm \Pi_2^A)$, respectively. In (8)-(10) the scalar amplitudes Π_i are the functions of kinematical invariants, i.e. $\Pi_i=\Pi_i(p_1^2,p_2^2,q^2)$.

The leading QCD term is a triangle quark-loop diagram in Fig. 1, for which we can write down the double dispersion representation at $q^2\le 0$

$$\Pi_i^{pert}(p_1^2,p_2^2,q^2) = -\frac{1}{(2\pi)^2}\int\frac{\rho_i^{pert}(s_1,s_2,Q^2)}{(s_1-p_1^2)(s_2-p_2^2)}ds_1ds_2 + \text{subtractions}, \qquad (11)$$

where $Q^2=-q^2\ge 0$. The integration region in (11) is determined by the condition

$$-1 < \frac{2s_1s_2+(s_1+s_2-q^2)(m_b^2-m_c^2-s_1)}{\lambda^{1/2}(s_1,s_2,q^2)\lambda^{1/2}(m_c^2,s_1,m_b^2)} < 1, \qquad (12)$$

where $\lambda(x_1,x_2,x_3)=(x_1+x_2-x_3)^2-4x_1x_2$. The expressions for spectral densities $\rho_i^{pert}(s_1,s_2,Q^2)$ are given in Appendix A.

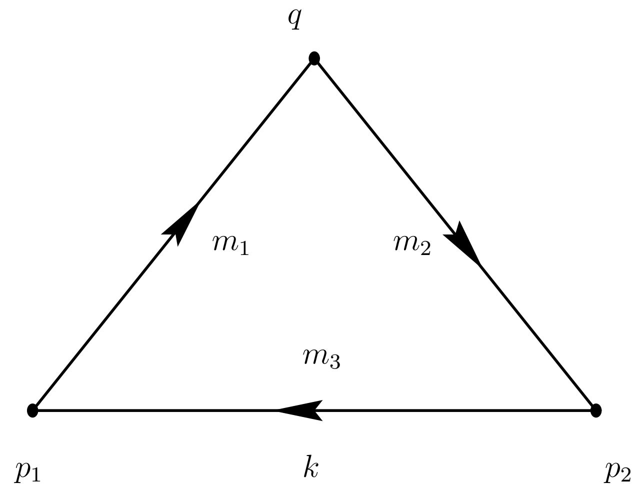

Figure 1: The triangle diagram, giving the leading perturbative term in the OPE expansion of three-point function.

Now let us proceed with the physical part of three-point sum rules. The connection to hadrons in the framework of QCD sum rules is obtained by matching the resulting QCD expressions of current correlators with the spectral representation, derived from a double dispersion relation at $q^2 \leq 0$.

$$\Pi_i(p_1^2, p_2^2, q^2) = -\frac{1}{(2\pi)^2} \int \frac{\rho_i^{phys}(s_1, s_2, Q^2)}{(s_1 - p_1^2)(s_2 - p_2^2)} ds_1 ds_2 + \text{subtractions}. \tag{13}$$

Assuming that the dispersion relation (13) is well convergent, the physical spectral functions are generally saturated by the ground hadronic states and a continuum starting at some effective thresholds s_1^{th} and s_2^{th}

$$\begin{aligned} \rho_i^{phys}(s_1, s_2, Q^2) &= \rho_i^{res}(s_1, s_2, Q^2) + \\ & \theta(s_1 - s_1^{th}) \cdot \theta(s_2 - s_2^{th}) \cdot \rho_i^{cont}(s_1, s_2, Q^2), \end{aligned} \tag{14}$$

where the resonance term is expressed through the product of leptonic constant and form factor for the transition under consideration, so that

$$\begin{aligned} \rho_i^{res}(s_1, s_2, Q^2) &= \langle 0|\bar{b}\gamma_5(\gamma_\mu)s|B_s(B_s^*)\rangle \, \langle B_s(B_s^*)|F_i(Q^2)|B_c\rangle \, \langle B_c|\bar{b}\gamma_5 c|0)\rangle \cdot \\ & (2\pi)^2\delta(s_1 - M_1^2)\delta(s_2 - M_2^2) + \text{higher state contributions}, \end{aligned} \tag{15}$$

where $M_{1,2}$ denote the masses of hadrons in the initial and final states. The continuum of higher states is modelled by the perturbative absorptive part of Π_i, i.e. by ρ_i. Then, the expressions for the form factors F_i can be derived by equating the representations for the three-point functions Π_i in (11) and (13), which means the formulation of sum rules.

For the heavy quarkonium $\bar{b}c$, where the relative velocity of quark movement is small, an essential role is taken by the Coulomb-like α_s/v-corrections. They are caused by the ladder diagram, shown in Fig. 2. It is well known that an account for this corrections in two-point sum rules numerically leads to a double-triple multiplication of Born value of spectral density [19, 21]. In our case it leads to the finite renormalization for ρ_i [13], so that

$$\rho_i^c = \mathbf{C}\rho_i, \tag{16}$$

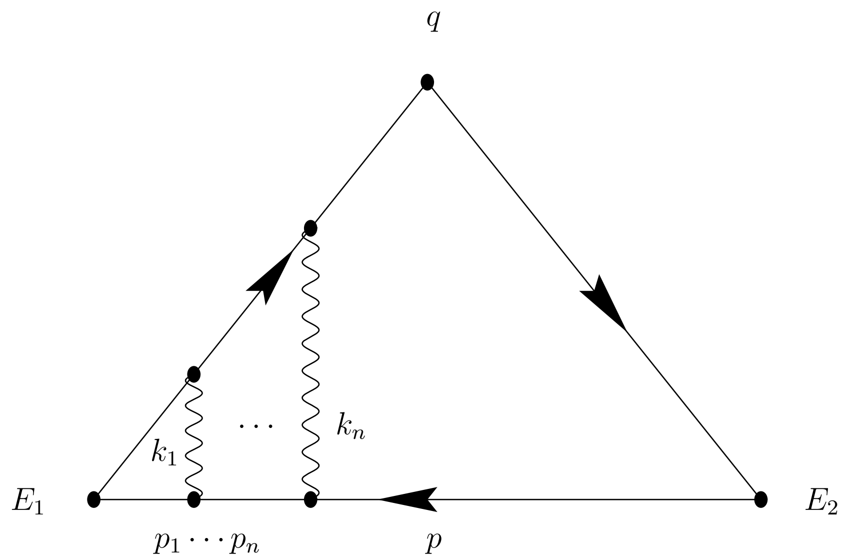

Figure 2: The ladder diagram of the Coulomb-like interaction.

$$\mathbf{C} = \frac{|\Psi^{C}_{\bar{b}c}(0)|}{|\Psi^{free}_{\bar{b}c}(0)|} = \sqrt{\frac{4\pi\alpha_s}{3v}(1-\exp\{-\frac{4\pi\alpha_s}{3v}\})^{-1}}, \tag{17}$$

where v is the relative velocity of quarks in the $\bar{b}c$-system,

$$v = \sqrt{1-\frac{4m_b m_c}{p_1^2-(m_b-m_c)^2}}. \tag{18}$$

To the moment, the procedure of calculations is completely described.

3 Numerical results on the form factors and the semileptonic decay widths

We evaluate the form factors in the scheme of spectral density moments. This scheme is not strongly sensitive to the value of the $\bar{b}c$-system threshold energy. In our calculations $E_c^{\bar{b}c} = 1.2$ GeV. The two-point sum rules for the B_c meson with account for the Coulomb-like corrections give $\alpha_s^c(\bar{b}c)$=0.45, which corresponds to f_{B_c}=400 MeV [21]. The quark masses are fixed by the calculations of leptonic constants f_Ψ and f_Υ in the same order over α_s. The requirement of stability in the sum rules including the contributions of higher excitations, results in quite an accurate determination of masses $m_c = 1.40 \pm 0.03$ GeV and $m_b = 4.60 \pm 0.02$ GeV, which are in a good agreement with the recent estimates in [22], where the quark masses free off a renormalon ambiguity were introduced. The values of leptonic constants f_Ψ, f_Υ linearly depend on the Coulomb-exchange α_s. We find $\alpha_s^c(\bar{c}c) \simeq 0.60$, $\alpha_s^c(\bar{b}b) \simeq 0.37$, which obey the remormalization group evolution with the appropriate scale prescription, depending on the quarkonium contents. In this way, we can extract the above values of $\alpha_s^c(\bar{b}c)$ and f_{B_c}. Note,

that the heavy $(Q_1\bar{Q}_2)$-quarkonia constants obey the scaling relation [21, 9]

$$\frac{f_n^2}{M_n}\left(\frac{M_n(m_1+m_2)}{4m_1m_2}\right)^2=\frac{c}{n}, \tag{19}$$

where n denotes the radial excitation number of nS-level, and c is independent of heavy quark flavors.

The leptonic constant for the B_s meson is extracted from the two-point sum rules. The Borel improved sum rules for the B meson leptonic constant [23] have the following form:

$$f_B^2 M_B e^{-\bar{\Lambda}(\mu)\tau}=K^2\frac{3}{\pi^2}C(\mu)\int\limits_0^{\omega_0(\mu)}d\omega\;\omega^2e^{-\omega\tau}+\langle\bar{q}q\rangle(1-\frac{m_0^2\;\tau^2}{16}+\frac{\pi^2\tau^4}{288}\langle\frac{\alpha_s}{\pi}G^2\rangle), \tag{20}$$

where we use $\langle\bar{q}q\rangle=-(0.23\cdot\mathrm{GeV})^3$, $m_0^2=0.8\ \mathrm{GeV}^2$, $\langle\frac{\alpha_s}{\pi}G^2\rangle=1.77\cdot10^{-2}\ \mathrm{GeV}^4$ as the central values, and M_B=5.28 GeV. The K-factor is due to α_s-corrections. We expect it is large, but we suppose the appearance of the same factor in evaluating the α_s-corrections to the heavy-light vertex in the triangle diagram. For this factor we have the following expression [23]:

$$K^2=\Big\{\int\limits_0^{E_c\tau}z^2e^{-z}dz\Big\}^{-1}\cdot\int\limits_0^{E_c\tau}z^2e^{-z}\Big\{1+\frac{2\alpha_s(\tilde{\Lambda})}{\pi}\left(\frac{13}{6}+\frac{2\pi^2}{9}-\ln\,z\right)\Big\}dz, \tag{21}$$

where we suppose that the scale $\tilde{\Lambda}$ is equal to 1.25 GeV. The dependence of K-factor on the Borel parameter τ and the threshold energy E_c is shown in Fig. 3. The K-factor is not sensitive to E_c changing in the range $1.0\div1.5$ GeV.

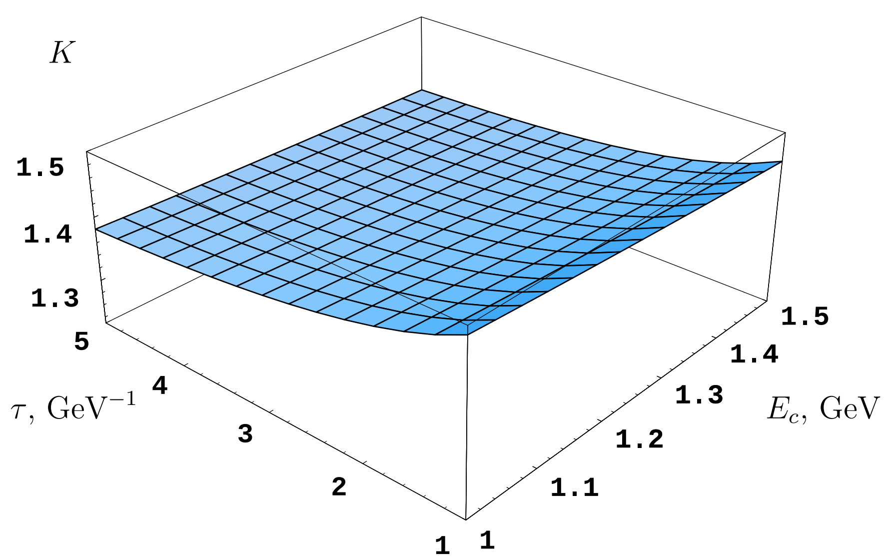

Figure 3: The K-factor dependence on the Borel parameter τ and the threshold energy E_c.

We see that NLO corrections to the leptonic constant are about 40%. Using the Padé approximation, we find that higher orders corrections can be about 30%. So, we hold the K factor in conservative

limits $1.4 \div 1.7$. It is quite reasonable to suppose its cancellation in evaluating the semileptonic form factors due to the renormalization of heavy-light vertex in the triangle diagram.

The contribution of quark condensate term is not sensitive to the variation of $\langle\bar{q}q\rangle$ in the limits from -$(0.23\cdot\mathrm{GeV})^3$ to -$(0.27\cdot\mathrm{GeV})^3$ (this variation corresponds to the renormalization group evolution and the insertion of α_s-corrections to this term, so that K can be putted as the overall factor).

In the limit of semi-local duality [24, 25] $\tau \to 0$ we get the relation: $\bar{\Lambda}(\mu) = \frac{3}{4}\,\omega_0(\mu)$ (the contribution of the quark condensate term to this equation is about $\sim 15\%$). We introduce the renormalization invariant quantities

$$\omega^{ren}_{0,dual} = C^{-1/3}(\mu)\;\omega_0(\mu), \qquad \bar{\Lambda}^{ren}_{dual} = \frac{3}{4}\;\omega^{ren}_{0,dual}.$$

For $\bar{\Lambda}^{ren}_{dual}$ we have $\bar{\Lambda}^{ren}_{dual} = M_B - m_b = 0.63$ GeV, and we obtain that in the semi-local duality the threshold energy $\omega^{ren}_{0,dual} = 0.84$ GeV. Neglecting the quark condensate term in the leptonic constant we have

$$f_B^2 M_B = K^2\frac{3}{\pi^2}(\omega^{ren}_{0,dual})^3. \tag{22}$$

Since in the three-point sum rules we use the scheme of moments and search for a stable region, in the general Borel scheme for f_B we have to consider the stability at $\tau \neq 0$ with the extended region of resonance contribution. We expect, that the sum rules in (20) with the redefined ω^{ren} and $\bar{\Lambda}^{ren}$, as mentioned, have a stability point at $\tau \sim \frac{1}{\bar{\Lambda}}$

$$f_B^2 M_B e^{-\bar{\Lambda}\tau} = K^2\frac{3}{\pi^2}\int\limits_0^{E_c} d\omega\;\omega^2 e^{-\omega\tau} + \langle\bar{q}q\rangle(1 - \frac{m_0^2\;\tau^2}{16} + \frac{\pi^2\tau^4}{288}\langle\frac{\alpha_s}{\pi}G^2\rangle), \tag{23}$$

where E_c is already not equal to $\omega^{ren}_{0,dual}$. Demanding a low deviation of $\bar{\Lambda}$ from $\bar{\Lambda}^{ren} = 0.63$ GeV, we find that sum rules in Eq.(23) can lead to the results, which are in a good agreement with the semi-local duality if $E_c = 1.1 \div 1.3$ GeV (see Fig. 4). Then the optimal value of Borel parameter $\tau = \tau_m \simeq 6.5$ GeV^{-1}. We write down

$$f_B^2 M_B e^{-\bar{\Lambda}\tau_m} = K^2\frac{3}{\pi^2}RE_c^3 + \langle\bar{q}q\rangle(1 - \frac{m_0^2\;\tau_m^2}{16} + \frac{\pi^2\tau_m^4}{288}\langle\frac{\alpha_s}{\pi}G^2\rangle), \tag{24}$$

where R denotes the average value of $e^{-\omega\tau_m}$. So, we find the $E_c^{3/2}$-dependence of $f_B\sqrt{M_B}$, whereas the contribution of condensate is numerically suppressed, as expected from the semi-local duality. The results of general Borel scheme calculations of $f_B\sqrt{M_B}$ ignoring the overall K-factor are presented in Fig. 4. We observe two stability regions. The stability region at $\tau = 2 \div 4$ corresponds to that of considered in [23]. The results for the leptonic constant f_B obtained from this region [23] is about 1.5 greater than the value obtained from the stability region at $\tau = 6 \div 7$. The second region appears only when we introduce the term with the product of quark and gluon condensates. The similar situation has been observed in the NRQCD sum rules for doubly heavy baryons [26]. The product of quark and gluon condensates was not taken into account in [23], and therefore, the intermediate stability point was observed only. Fixing the optimal values of $f_B^2 M_B$ in Eq.(24) from Fig. 4, we can invert the sum rules to study the dependence of $\bar{\Lambda}$ on τ, as shown in Fig. 5, where the optimal values of $\bar{\Lambda}$ agree with the semi-local duality and the estimate $\bar{\Lambda}^{ren} = M_B - m_b$. Note that the intermediate stability point $\tau \sim 4$ exhibit a low variation of $\bar{\Lambda}$ close to 0.4 GeV, which was obtained in [26, 23], and usually given by the potential models (see, for instance, [27]).

The calculated physical quantity should be independent of parameters in the sum rule scheme. However, we truncate both the operator product expansion and the perturbative series for the Wilson

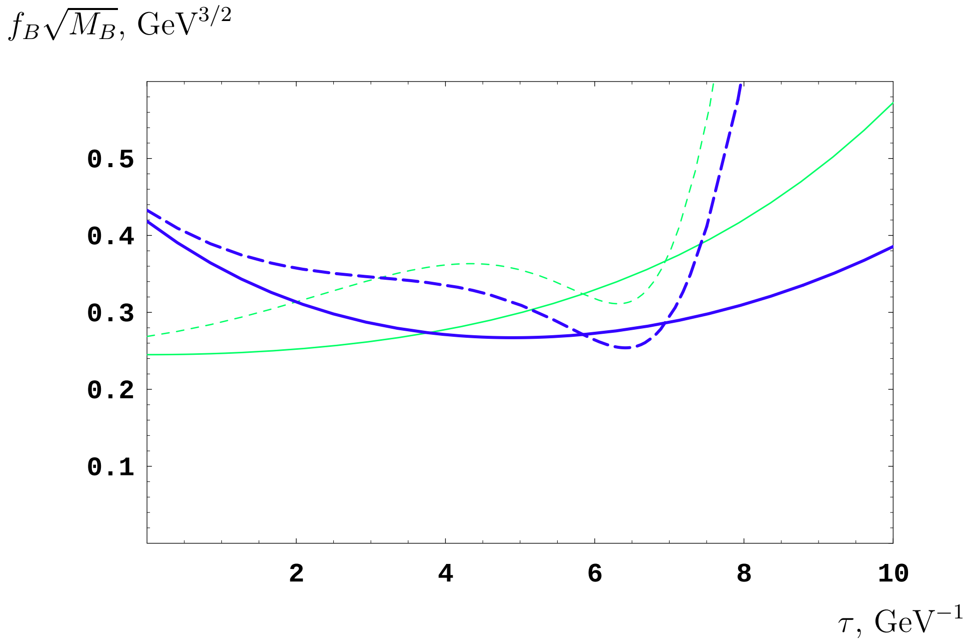

Figure 4: f_B in the semi-local duality sum rules (pale curves) and in the general Borel scheme. Solid lines correspond to the sum rules without condensate terms, dashed lines correspond to the accounting for the condensates contributions.

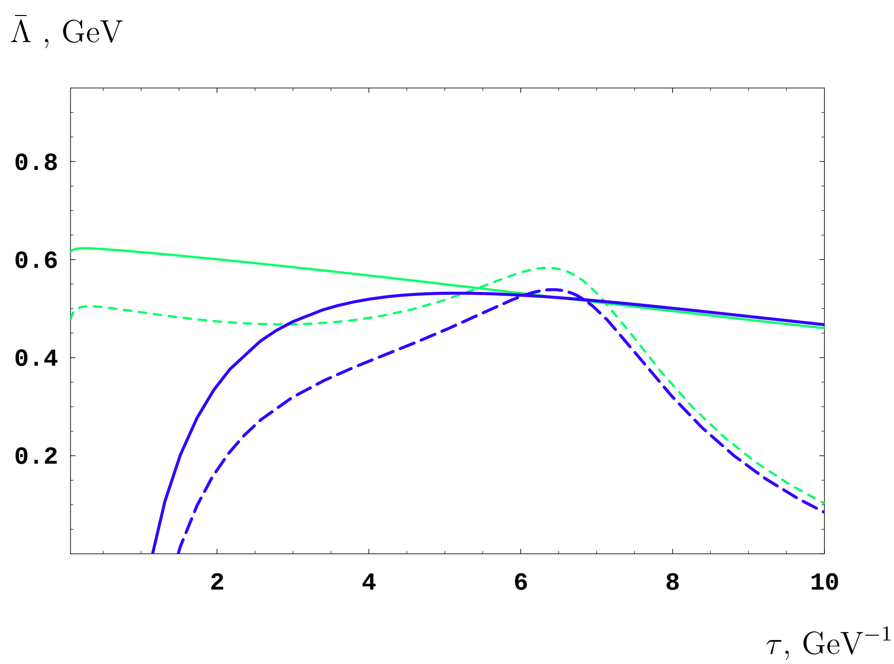

Figure 5: The dependence of $\bar{\Lambda}$ on τ with the fixed values of $f_B\sqrt{M_B}$, which correspond to the stability regions at $\tau = 6 \div 7$ GeV^{-1} (The notations are the same as in Fig. 4).

coefficients by fixed orders in the opreator dimension and α_s, respectively. This fact leads to that the results depend on the scheme parameters, say, the Borel variable. Moreover, the physical part of sum rules is modelled by the contributions of resonances and a continuum term starting at a threshold, that introduces the dependence on the threshold value and suggests that the stability of results can be improved by the terms of excited resonaces in addition to the ground state.

Let us, at first, consider the results on the leptonic constant in the limit of semi-local duality, which requires the stability at $\tau \to 0$ and corresponds to the duality region containing the ground state, only. The sum rules show that the condensate contributions are given by the polinomials over τ, and the leading correction at $\tau \to 0$ is the term with the quark condensate. Then, we expect that the region of stability will extend at $\tau > 0$ if we will add the higher condensates with the appropriate ratios of their values. The central values of condensates as mentioned above correspond to the results shown in Fig. 4. The stability of semi-local sum rules can be improved by a variation of $\bar{\Lambda}$ and ω_0, which is not important for the current discussion. In order to clarify this statement we present the results of semi-local sum rules for the leptonic constant of B meson in Fig. 6, where we put $\bar{\Lambda} = 0.54$ GeV. Note, that the value of leptonic constant is the same as we have found in the pertubative limit of semi-local duality. Then, we state that the value of leptonic constant obtained at $\tau \to 0$ agrees with the value corresponding to the stability at $\tau \approx 7$ GeV^{-1}, i.e. in the second point of local extremum in Fig. 4. In order to confirm, we present also the result for the other ratio of condensate values (the lower value of mixed condensate and the upper value of gluon condensate in the regions mentioned below) in Fig. 7. We see that the semi-local duality is completely broken at unappropriate choice of condensate values.

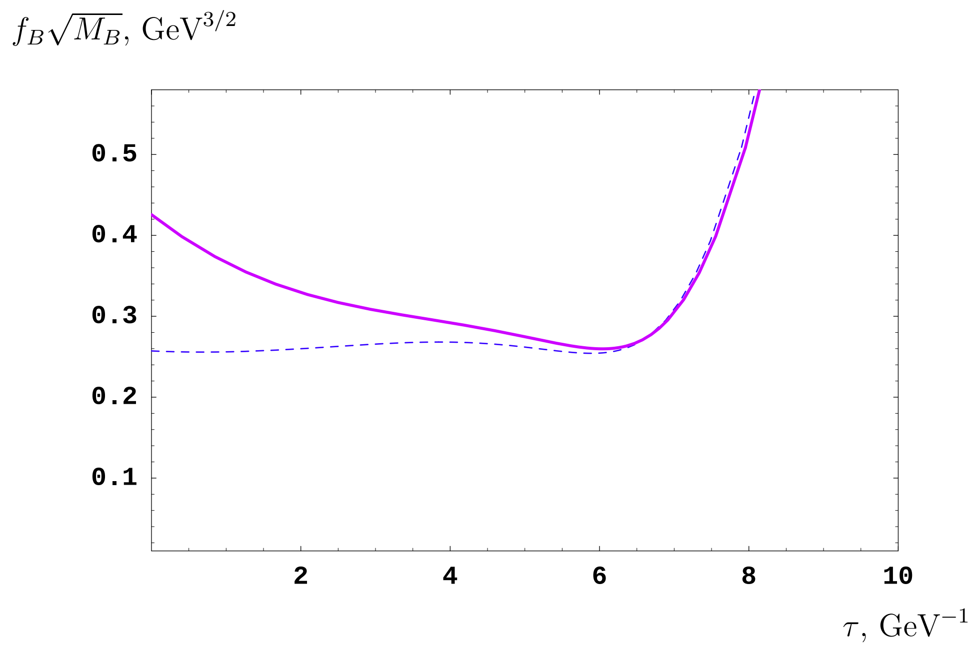

Figure 6: f_B in the semi-local duality sum rules (dashed curve) and in the general Borel scheme (solid line) with the corrected value of $\bar{\Lambda}$, which improves the stability of result obtained in the semi-local duality.

Second, we study the general Borel sum rules in the scheme, where we do not require the stability at $\tau \to 0$, which means that we extend the duality region to incorporate possible excitation contributions.

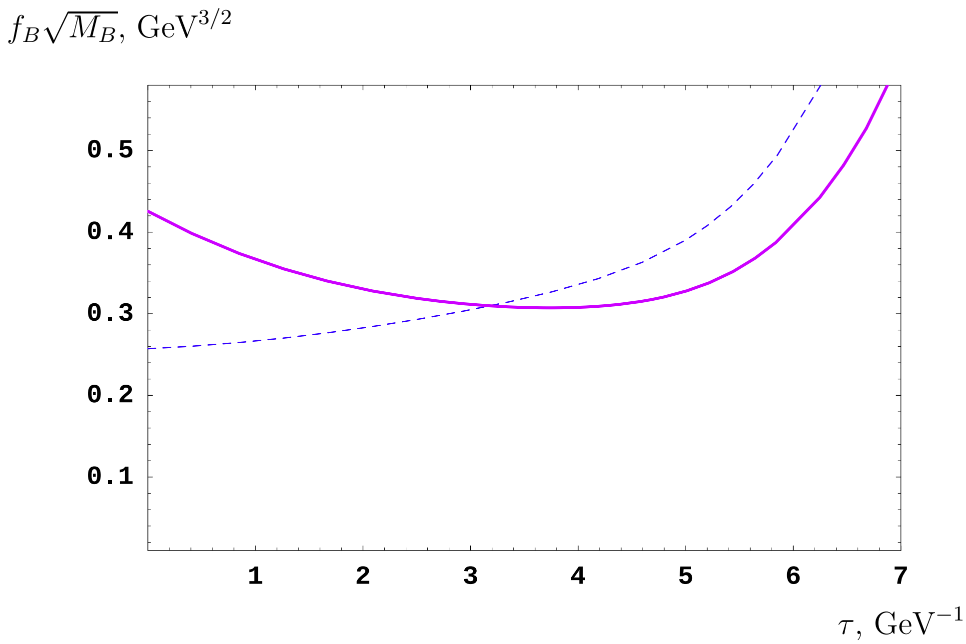

Figure 7: f_B in the semi-local duality sum rules (dashed curve) and in the general Borel scheme (solid line) with the condensate values, which destroy the semi-local duality.

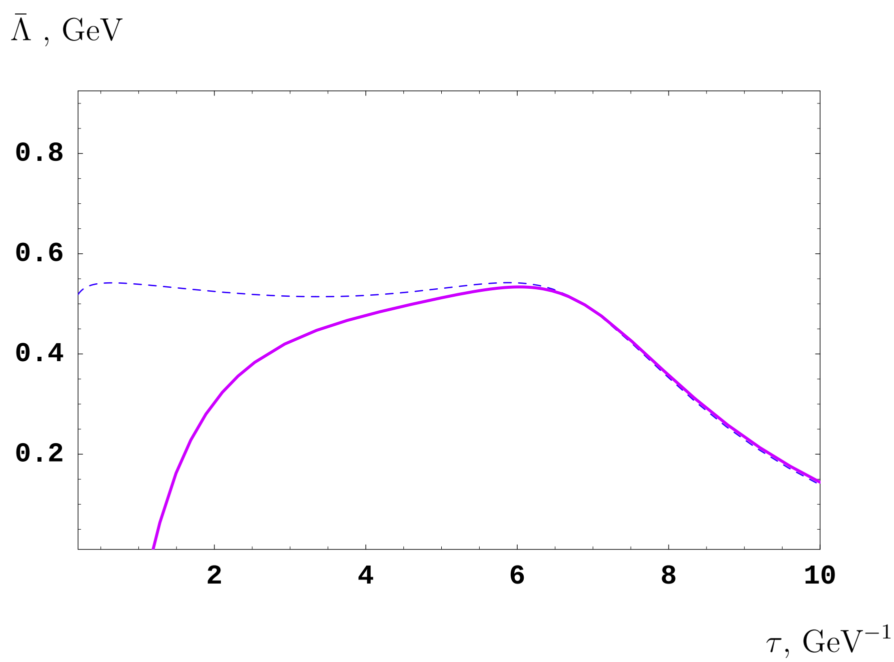

Figure 8: The dependence of $\bar{\Lambda}$ on τ with the fixed values of $f_B\sqrt{M_B}$, which correspond to the stability regions at $\tau = 6 \div 7$ GeV^{-1} in Fig. 6.

Then, we expect that the estimate of leptonic constant should result in the same value obtained in the semi-local duality. We see that this can be reached at the same values of condensates involved as well as in the same region of τ, which is again given by the second extremum (see Figs. 4 and 6). Certainly, the extension of duality region leads to a failure of stability at low τ, which is caused by additional terms given by the excitations beyond the control of the method.

At higher values of τ the stability would be reached by introduction of higher condensates contributing as the polinomials of higher powers. Of course, we can achieve the better stability in the general Borel scheme by adjusting the condensate values, but this will destroy the semi-local duality, that indicates the divergency of the method, while the convergency demands an appropriate choice of condensate values as we put.

The criterion on the covergency of both the semi-local duality and general Borel sum rules was ignored in [23], where the greater value of leptonic constant was obtained (see Fig. 7).

The same notes can be done in the discussion on $\bar{\Lambda}$. For the sake of comparison, we present the results in Figs. 5 and 8, corresponding to the inverted sum rules for the leptonic constant given in Figs. 4 and 6.

The physical meaning of gluon condensate as it stands in the operator product expansion taken between the observed states is independent of the scheme of calculations. In the sum rules the gluon condensate contributes in the region, where the excited states are suppressed, while the ground state corresponding to the current under consideration dominates. So, the condensate essentially determines the binding energy of quarks in the hadron containing the heavy quark. The definition of sum rules scheme includes the parameter determining the threshold energy of continuum contribution in addition to the resonance term. So, the difference between the schemes of semi-local duality and usual Borel representation is due to the variation of duality region. So, the semi-local duality means the duality for the region contaning the ground state only, while the usual Borel (or moment) scheme explore the extended region containing several hadronic states: the ground state and its excitations. However, the gluon condensate essentially contributes at the scheme parameters, when the excitations are suppressed. Thus, its value is the same for both schemes used, i.e. for the semi-local duality and in the Borel scheme. The value of gluon condensate has been varied in the range $\langle\frac{\alpha_s}{\pi}G^2\rangle = (1.5 \div 2)\cdot 10^{-2}$ GeV^4. As for the mixed condesate we have used the range $m_0^2 = (0.72 \div 0.88)$ GeV^2. The variation of these parameters does not change the qualitative picture for the lepton constant as it has been discussed, while the numerical uncertainty of its value is less than 7%.

Numerically, multiplying the result taken from Fig. 4, by the K-factor we find the value $f_B = 140 \div 170$ MeV, which is in a good agreement with the recent lattice results [28] and the estimates in the QCD SR by other authors [29]. So, we can conclude that the $1/m_b$-corrections are not valuable for f_B. The uncertainty of estimates is basically connected with the higher orders in α_s. For the vector B^* meson constant f_{B^*} we put $\frac{f_{B^*}}{f_B} = 1.11$ (see [30, 29]).

For the leptonic constant of B_s meson we explore the following relation $\frac{f_{B_s}}{f_B} = 1.16$, which expresses the flavor SU(3)-symmetry violation for B mesons [25].

Remember, in sum rules the heavy quark masses are fixed by the two-point sum rules for bottomonia and charmonia with the precision of 20 MeV. In our consideration the quark masses are equal to m_b=4.6 GeV, m_c=1.4 GeV, and we use m_s=0.15 GeV, which agrees with the various estimates [31]. The uncertainties in the values of form factors are basically determined by the variation of b-quark mass, while changing the other quark masses in the ranges $m_s = 0.14 \div 0.16$ GeV and $m_c = 1.35 \div 1.45$ GeV, results in the uncertainty less than 2%. In Figs. 9, 10 and 11 we present the results in the scheme of spectral density moments.

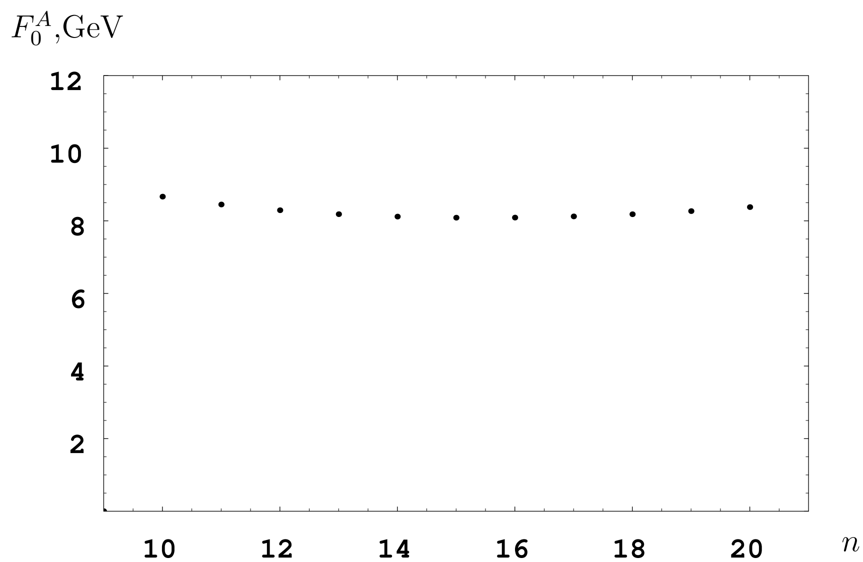

Figure 9: F_0^A in the scheme of the spectral density moments; n is the number of moment with respect to the square of momentum in the $\bar{b}c$ channel. The number of moment with respect to the square of momentum in the $\bar{b}s$ channel is equal to 1 at $E_c = 1.2$ GeV.

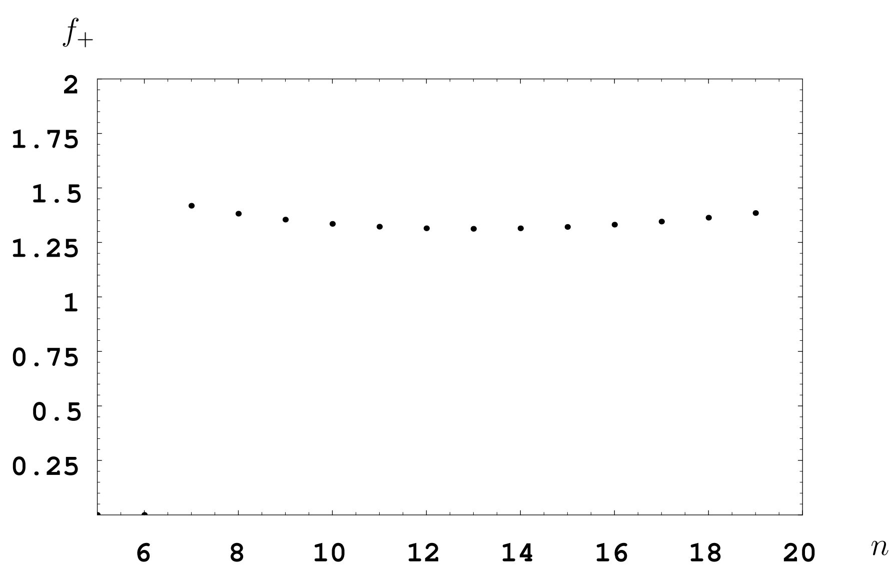

Figure 10: f_+ in the scheme of the spectral density moments; n is the number of moment with respect to the square of momentum in the $\bar{b}c$ channel. The number of moment with respect to the square of momentum in the $\bar{b}s$ channel is equal to 1 at $E_c = 1.2$ GeV.

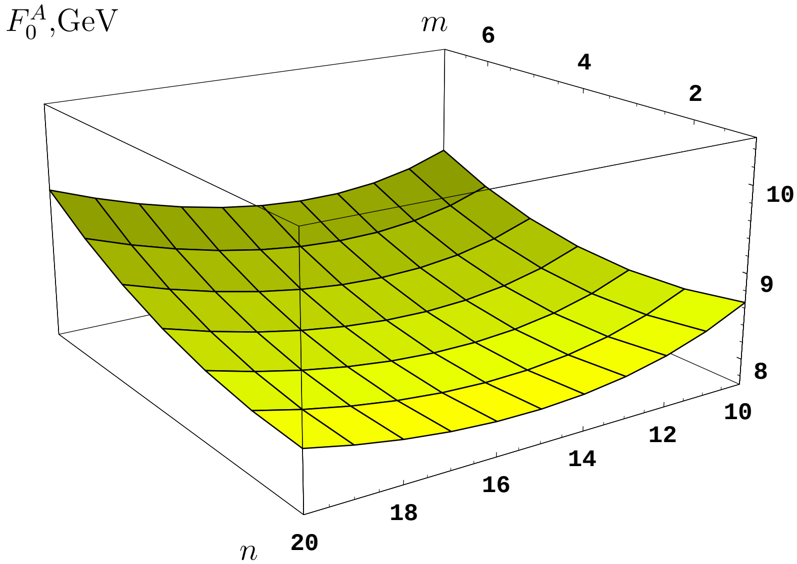

Figure 11: F_0^A in the scheme of the spectral density moments; n is the number of moment with respect to the square of momentum in the $\bar{b}c$ channel, m is the number of moment with respect to the square of momentum in the $\bar{b}s$ channel at $E_c = 1.2$ GeV.

We have investigated the dependence of form factors on the $\bar{b}s$ threshold energy of continuum in the range $E_c = 1.1 \div 1.3$ GeV. The characteristic forms of this dependence are shown in Figs. 12 and 13. We see that the optimal choice for the $\bar{b}s$ system threshold energy is 1.2 GeV. In Table 1 we present the results of sum rules for the form factors in comparison with estimates in the framework of potential models [9, 32]. We see a good agreement of estimates in the QCD sum rules with the values in the quark model. For the sake of completeness the quark model expressions for the form factors are given in Appendix B.

Method	f_+	f_-	F_V, GeV^{-1}	F_0^A, GeV	F_+^A, GeV^{-1}	F_-^A, GeV^{-1}
This paper	1.3	-5.8	1.1	8.1	0.2	1.8
Potential model [9]	1.1	-5.9	1.1	8.2	0.3	1.4

Table 1: The form factors of B_c decay modes into the B_s and B_s^* mesons at $q^2 = 0$.

In [10] the form factors were derived using the similar SR technique but without the Coulomb-like corrections in the $\bar{b}c$ system, which enhance the form factors about three times, as we have found.

Let us discuss the uncertainties in the sum rules and other approaches. So, the potential models [8, 15] with similar choices of parameters result in the form factor values, which are slightly model-dependent. The corresponding accuracy is about 10%. Then, we expect that the potential models give good reference points for the appropriate numerical values.

The accuracy of sum rules under consideration is basically determined by the variation of heavy quark masses. Indeed, the significant α_s correction to the leptonic constant of B_s meson should cancel

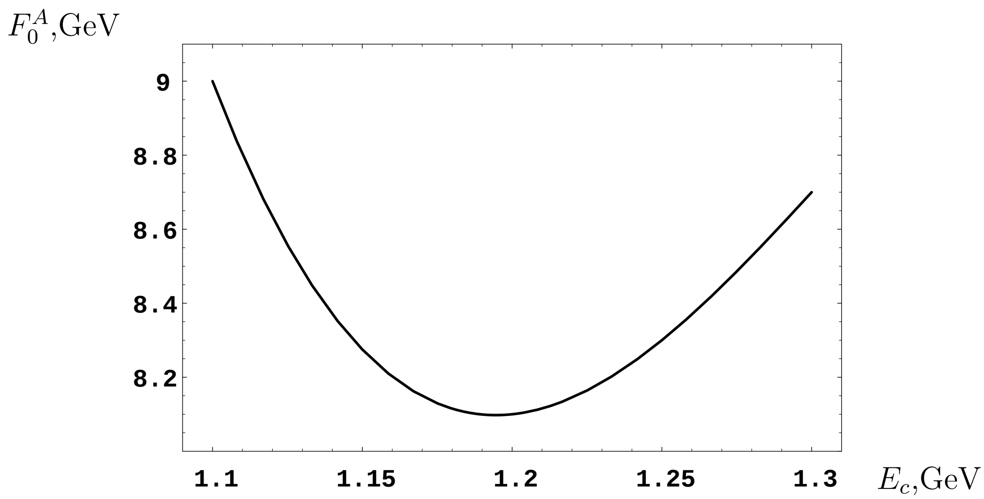

Figure 12: The dependence of F_0^A on the $\bar{b}$s threshold energy E_c, determining the region of resonance contribution.

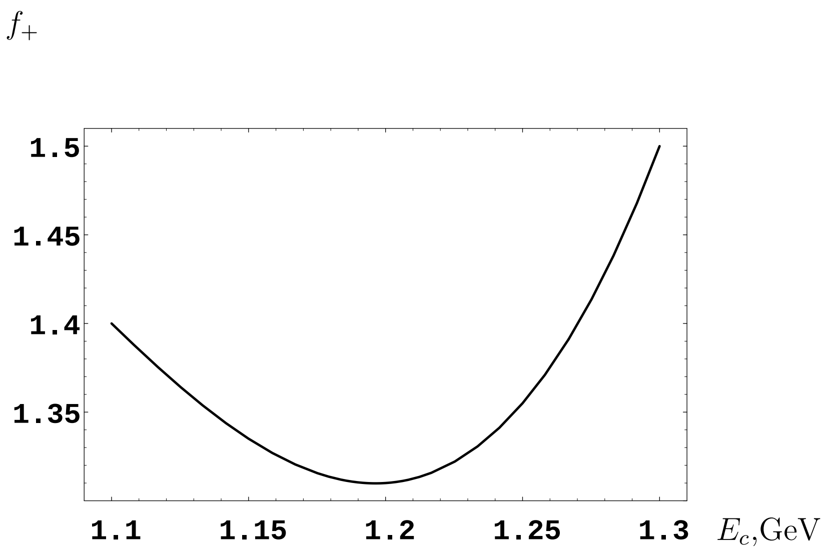

Figure 13: The dependence of f_+ on the $\bar{b}$s threshold energy E_c, determining the region of resonance contribution.

the same factor for the renormalization of quark-meson vertex in the triangle diagram. The dependence on the choice of threshold energy in the $\bar{b}s$-channel can be optimized and, hence, minimized. The variation of threshold energy in the $\bar{b}c$-channel give the error less than 1%. The effective coulomb constant is fixed from the two-point sum rules for the heavy quarkonium, and its variation is less than 2%, which gives the same uncertainty for the form factors. The heavy quark masses are determined by the two-point sum rules for the heavy quarkonia, too. However, their variations result in the most essential uncertainty. Summing up all of mentioned variations we estimate $\delta f/f \simeq 5\%$.

The enhancment of form factors in the decays of heavy quarkonium considered in the framework of sum rules was also found in the decays $B_c \to \psi l \nu$ [12, 13]. It is important to stress that the current consideration removes the contradiction between the estimates in OPE, potential models and the values in the SR. We have calculated the form factors in the SR consistent with the values in the OPE and potential models.

We use the three-point sum rules to determine the dependence on q^2 of form factors in vicinity of $q^2 = 0$, where the method works even in the approximation by a bare quark loop [20]. Naively, we expect a simple pole form

$$f(q^2) = \frac{f(0)}{1 - \frac{q^2}{M_{pole}^2}}, \tag{25}$$

so that the first derivative $f'(0) = f(0)\frac{1}{M_{pole}^2}$ can be evaluated in the framework of sum rules to estimate the "pole" mass M_{pole}, which may deviate from the value of physical mass of corresponding bound state. Of course, the bare quark loop approximation cannot be justified at $q^2 \sim m_{pole}^2$, while the three-point sum rules in the form of double dispersion relation cannot be explored in the region of resonances at $q^2 > 0$ because of so-called "non–Landau" singularities indicating the presence of strongly bound states in the q^2-channel. That is why we calculate the form factors as well as their Borel transforms at $q^2 < 0$.

Numerically, we have found M_{pole}=1.3÷1.4 GeV for the form factors with the B_s^* decay modes, and M_{pole}=1.8÷1.9 GeV for the decay form factors with the B_s modes.

The semileptonic widths are presented in Table 2. We have supposed the quark mixing matrix element $|V_{cs}|$=0.975 [31]. The mesons masses are equal to M_{B_c}=6.25 GeV, M_{B_s}=5.37 GeV, $M_{B_s^*}$=5.41 GeV [33].

mode	Γ, 10^{-14} GeV	BR, %
$B_s e^+ \nu_e$	5.8	4.0
$B_s^* e^+ \nu_e$	7.2	5.0

Table 2: The widths of semileptonic B_c decay modes and the branching fractions calculated at $\tau_{B_c} =$ 0.46 ps.

These results agree with the values obtained in the framework of covariant quark model [9]: $\Gamma(B_s e^+ \nu_e)$=$4.7 \cdot 10^{-14}$ GeV, $\Gamma(B_s^* e^+ \nu_e)$=$7.4 \cdot 10^{-14}$ GeV, as we could expect looking at Table 1.

4 The symmetry relations

At the recoil momentum close to zero, the heavy quarks in both the initial and final states have small relative velocities inside the hadrons, so that the dynamics of heavy quarks is essentially nonrelativistic. This allows us to use the combined NRQCD/HQET approximation in the study of mesonic form factors. The expansion in the small relative velocities to the leading order leads to various relations between the different form factors. Solving these relations results in the introduction of an universal form factor (an analogue of the Isgur-Wise function) at $q^2 \to q^2_{max}$.

We consider the soft limit

$$\begin{aligned} v_1^\mu &\neq v_2^\mu, \\ w = v_1 \cdot v_2 &\to 1, \end{aligned} \tag{26}$$

where $v^\mu_{1,2} = p^\mu_{1,2}/\sqrt{p^2_{1,2}}$ are the four-velocities of heavy mesons in the initial and final states. The study of region (26) is reasonable enough, because in the rest frame of B_c meson ($p_1^\mu = (\sqrt{p_1^2}, \vec{0})$), the four-velocities differ only by a small value $|\vec{p}_2|$ ($p_2^\mu = (E_2, \vec{p}_2)$, whereas their scalar product w deviates from unity only due to a term, proportional to the square of $|\vec{p}_2|$: $w = \sqrt{1 + \frac{|\vec{p}_2|^2}{p_2^2}} \sim 1 + \frac{1}{2}\frac{|\vec{p}_2|^2}{p_2^2}$. Thus, in the linear approximation at $|\vec{p}_2| \to 0$, relations (26) are valid and take place. Here we would like to note, that (26) generalizes the investigation of [14], where the case of $v_1 = v_2$ was considered. This condition severely restricts the relations of spin symmetry for the form factors and, as a consequence, it provides a single connection between the form factors. In the soft limit of zero recoil we find

$$\tilde{v}_3^\mu = -\frac{1}{2}(v_1^\mu + v_2^\mu) \tag{27}$$

for the four-velocity of spectator b-quark, and

$$\tilde{v}_1^\mu = v_1^\mu + \frac{m_3}{2m_1}(v_1 - v_2)^\mu \tag{28}$$

for the decaying c-quark. The matrix element of $J_\mu = \bar{Q}_1 \Gamma_\mu q_2$ with the spin structure $\Gamma_\mu = \{\gamma_\mu, \gamma_5\gamma_\mu\}$ has the form

$$\begin{aligned} \langle H_{Q_1\bar{Q}_3}|J_\mu|H_{q_2\bar{Q}_3}\rangle &= tr[\Gamma_\mu(1+\tilde{v}_1^\mu\gamma_\mu)\Gamma_1(1+\tilde{v}_3^\nu\gamma_\nu)\cdot \\ &\qquad \Gamma_2\rho_{light}]\cdot h, \end{aligned} \tag{29}$$

where Γ_1 determines the spin state in the heavy meson $Q_1\bar{Q}_3$ (in our case it is pseudoscalar, so that $\Gamma_1 = \gamma_5$), Γ_2 determines the spin wave function of quarkonium in the final state $\Gamma_2 = \{\gamma_5, \epsilon^\mu\gamma_\mu\}$ for the pseudoscalar and vector states, respectively ($H = P, V$). The 'propagator of the light quark'[5] is taken in a general form

$$\rho_{light} = 1 + B(\not{v}_2 - \not{v}_1) + C(\not{v}_2 + \not{v}_1) + D\not{v}_2\not{v}_1, \tag{30}$$

where B, C, D are the functions of w. The quantity h in (29) at $w \to 1$ is an universal factor independent of the spin state of meson. So, for the form factors, discussed in our paper, we have

$$\begin{aligned} \langle P_{Q_1\bar{Q}_3}|\bar{Q}_1\gamma^\mu Q_3|P_{q_2\bar{Q}_3}\rangle &= (c_1^P\cdot v_1^\mu + c_2^P\cdot v_2^\mu)\cdot h, &(31)\\ \langle P_{Q_1\bar{Q}_3}|\bar{Q}_1\gamma^\mu Q_3|V_{q_2\bar{Q}_3}\rangle &= ic_V\cdot\epsilon^{\mu\nu\alpha\beta}\epsilon_\nu v_{1\alpha}v_{2\beta}\cdot h, &(32)\\ \langle P_{Q_1\bar{Q}_3}|\bar{Q}_1\gamma_5\gamma^\mu Q_3|V_{q_2\bar{Q}_3}\rangle &= (c_\epsilon\cdot\epsilon^\mu + c_1\cdot v_1^\mu(\epsilon\cdot v_1) + c_2\cdot v_2^\mu(\epsilon\cdot v_1))\cdot h, &(33) \end{aligned}$$

[5] More strictly, we determine the spin structure of matrix element and the term given by light degrees of freedom.

where

$$
\begin{aligned}
c_\epsilon &= -2, \\
c_V &= -1 - \tilde{B} - \frac{m_3}{2m_1}, \\
c_1^P &= 1 - \tilde{B} + \frac{m_3}{2m_1}, \\
c_2^P &= 1 + \tilde{B} - \frac{m_3}{2m_1},
\end{aligned}
\tag{34}
$$

and $\tilde{B} = \frac{B-2D}{1+C}$. The rest coefficients $c_{1,2}$ depend on the C and D parameters. We have the symmetry relations for the following form factors[6]:

$$
\begin{aligned}
f_+(c_1^P \cdot \mathcal{M}_2 - c_2^P \mathcal{M}_1) - f_-(c_1^P \cdot \mathcal{M}_2 + c_2^P \cdot \mathcal{M}_1) &= 0, \\
F_0^A \cdot c_V - 2c_\epsilon \cdot F_V \mathcal{M}_1 \mathcal{M}_2 &= 0, \\
F_0^A c_1^P + c_\epsilon \cdot \mathcal{M}_1 (f_+ + f_-) &= 0,
\end{aligned}
\tag{35}
$$

where $\mathcal{M}_1 = m_1 + m_3$, $\mathcal{M}_2 = m_2 + m_3$. Equating the second relation in (35), for example, we obtain

$$
\tilde{B} = -\frac{2m_1 + m_3}{2m_1} + \frac{4m_3(m_1+m_3)F_V}{F_0^A} \simeq 10.0, \tag{36}
$$

where all form factors are taken at q^2_{max}. Substituting $\tilde{B}$ in first and third relations, we get $f_+ \simeq 2.0$ and $f_- \simeq -8.3$. These values have to be compared with the corresponding form factors obtained in the QCD sum rules: $f_+(q^2_{max}) = 1.8$ and $f_-(q^2_{max}) = -8.1$, where we suppose the pole like behaviour of form factors (see Eq.(25)). Thus, we find that in the QCD sum rules, relations (35) are valid with the accuracy better than 10% at $q^2 = q^2_{max}$. The deviation could increase at $q^2 < q^2_{max}$ because of variations in the pole masses governing the evolution of form factors. However, in $B_c^+ \to B_s^{(*)} l^+ \nu$ decays the phase space is restricted, so that the changes of form factors are about 50%, while their ratios develop more slowly.

5 Nonleptonic decays and the lifetime

The hadronic decay widths can be obtained on the basis of assumption on the factorization for the weak transition between the quarkonia and the final two-body hadronic states. For the dominant nonleptonic decay modes $B_c^+ \to B_s^{(*)} \pi^+(\rho^+)$ the effective Hamiltonian can be written down as

$$
H_{eff} = \frac{G_F}{2\sqrt{2}} V_{cs} V_{ud}^* \{C_+(\mu) O_+ + C_-(\mu) O_-\}, \tag{37}
$$

where

$$
O_\pm = (\bar{u}_i \gamma_\nu (1-\gamma_5) d_i)(\bar{s}_j \gamma^\nu (1-\gamma_5) c_j) \pm (\bar{u}_i \gamma_\nu (1-\gamma_5) d_j)(\bar{s}_i \gamma^\nu (1-\gamma_5) c_j), \tag{38}
$$

where i, j run over the colors. The factors $C_\pm(\mu)$ account for the strong corrections to the corresponding four-fermion operators caused by hard gluons. The review on the evaluation of $C_\pm(\mu)$ can

[6]To remove an error in [13] the analogous second relation for the $B_c \to J/\Psi(\eta_c)$ transition should have the missed factor 2 in front of c_ϵ.

be found in [34]. In the present paper, dealing with the QCD sum rules in the leading order over α_s, we explore the $C_\pm(\mu)$-evolution to the leading log accuracy. The $B_c^+ \to B_s\pi^+$ amplitude, for example, takes the form

$$A(B_c^+ \to B_s\pi^+) = \frac{G_F}{\sqrt{2}} V_{cs}V_{ud}a_1(\mu)\langle\pi^+|\bar{u}\gamma_\nu(1-\gamma_5)d|0\rangle\langle B_s|\bar{s}\gamma^\nu(1-\gamma_5)c|B_c\rangle, \tag{39}$$

where $a_1(\mu) = \frac{1}{2N_c}(C_+(\mu)(N_c+1) + C_-(\mu)(N_c-1))$ at $N_c = 3$ being the number of colors. In our calculations we put the following light meson parameters: m_{π^+}=0.14 GeV, m_{ρ^+}=0.77 GeV, f_{π^+}=0.132 GeV, f_{ρ^+}=0.208 GeV. The results are collected in Table 3.

mode	Γ, 10^{-14} GeV	BR, %
$B_s\pi^+$	15.8 a_1^2	17.5
$B_s\rho^+$	6.7 a_1^2	7.4
$B_s^*\pi^+$	6.2 a_1^2	6.9
$B_s^*\rho^+$	20.0 a_1^2	22.2

Table 3: The widths of dominant nonleptonic B_c decay modes due to $c \to s$ transition and the branching fractions calculated at $\tau_{B_c} = 0.46$ ps. We put a_1=1.26.

It is worth noting that the sum of widths for transitions $B_c^+ \to B_s(B_s^*)\pi^+(\rho^+)$ is 10% larger than the width for the transition $B_c^+ \to B_s(B_s^*) + \mathit{light\ hadrons}$, which is calculated using the simple formula

$$\Gamma[B_c^+ \to B_s(B_s^*) + \mathit{light\ hadrons}] = N_c\ a_1^2(\mu)\ \Gamma[B_c^+ \to B_s(B_s^*)e^+\nu_e],$$

where we neglect the contributions given by the modes with the factor a_2 instead of a_1. In addition, the deviation between these estimates can be caused by the corresponding 'bag' factor appearing in the formulation of factorization approach and vacuum saturation in the connection of leptonic form factors to the hadronic ones. The modern lattice estimates show that the 'bag' parameters are about 7% less than 1 [28].

In the parton approximation we could expect

$$\Gamma[B_c^+ \to B_s^{(*)} + \mathit{light\ hadrons}] = (2C_+^2(\mu) + C_-^2(\mu))\Gamma[B_c^+ \to B_s^{(*)}e^+\nu_e],$$

which results in the estimate very close to the value obtained as the sum of exclusive modes at $\mu > 0.9$ GeV. The deviation between these two estimates slightly increase at $\frac{m_c}{2} < \mu < 0.9$ GeV. Concerning the comparison of hadronic width summing up the exclusive decay modes with the estimate based on the quark-hadron duality, we insist that the deviation between these two estimates is less that 10% and, hence, it cannot be treated as an essential argument against the validity of our calculations.

We estimate the lifetime using the fact that the dominant modes of the B_c meson decays are the $c \to s$, $b \to c$ transitions with the $B_s^{(*)}$ and J/ψ, η_c final states respectively, and the electroweak annihilation [7].

We stress that in the $B_c \to B_s$ decays caused by the weak decays of charmed quark the possible hadronic final states are the charged mesons π, ρ and K or the multi-particle states like $\pi\pi$, $\pi\pi\pi$

[7] The $\bar{b} \to \bar{c}c\bar{s}$ transition is negligibly small in the B_c decays because of destructive Pauli interference for the charmed quark in the initial state and the product of decay [6].

or $K\pi$. First, the states with the kaon are suppressed by the Cabibbo angle, and we neglect their contributions in the total nonleptonic width of B_c (the corresponding error of estimates is about 4%). The method for the calculation of multi-particle branching fractions was offered by Bjorken in his pioneering paper devoted to the decays of hadrons containing heavy quarks [35]. He supposed a simple relation for the yields of pions, as given by the Poisson distribution with the average value of pions determined by the energy release. The Bjorken's model of multi-particle yields in the decays does not distinguish the resonant and continuum final states as well as B_s and B_s^*. So, we check that the ratio of $R_{2\pi} = \Gamma(B_c^+ \to B_s^{(*)}\pi^+)/\Gamma(B_c^+ \to B_s^{(*)}\rho^+) \approx 0.82$ calculated in the framework of sum rules is close to the estimate $R_{2\pi} = \Gamma(B_c^+ \to B_s^{(*)}\pi^+)/\Gamma(B_c^+ \to B_s^{(*)}\pi^+\pi^0) \approx 0.85$ given by Bjorken. Then, we see that the non-resonant multi-particle states are suppressed in comparison with the resonance yields. The same fact can be found, once we consider the $K\pi$, $K\pi\pi$ and $K^*\pi$, $K\rho$ branching fractions in the decays of D mesons as measured experimentally. This consideration confirms us that we take into account all of significant nonleptonic decay modes of B_c. In order to estimate the contribution of non-resonant 3π modes of B_c decays into $B_s^{(*)}$ we use the Bjorken's technique, i.e. the Poisson distribution with the average value corrected to agree with the non-resonant 3π-modes in the decays of D mesons. We have chosen the following branching ratios: $\mathrm{BR}(D^+ \to K^-\pi^+\pi^+) = 9.0 \pm 0.6\%$ and $\mathrm{BR}(D^+ \to K^-\pi^+\pi^+\pi^0|_{\mathrm{non-resonant}}) = 1.2 \pm 0.6\%$. So, we have found $\bar{n} \approx \frac{1}{8}$, which means that $\mathrm{BR}(B_c^+ \to B_s^{(*)}(3\pi)^+) \approx 0.2\%$, while $\mathrm{BR}(B_c^+ \to B_s^{(*)}(2\pi)^+|_{\mathrm{non-resonant}}) \approx 3\%$. We see that the neglected modes contribute to the total width of B_c as a small fraction in the limits of uncertainty envolved.

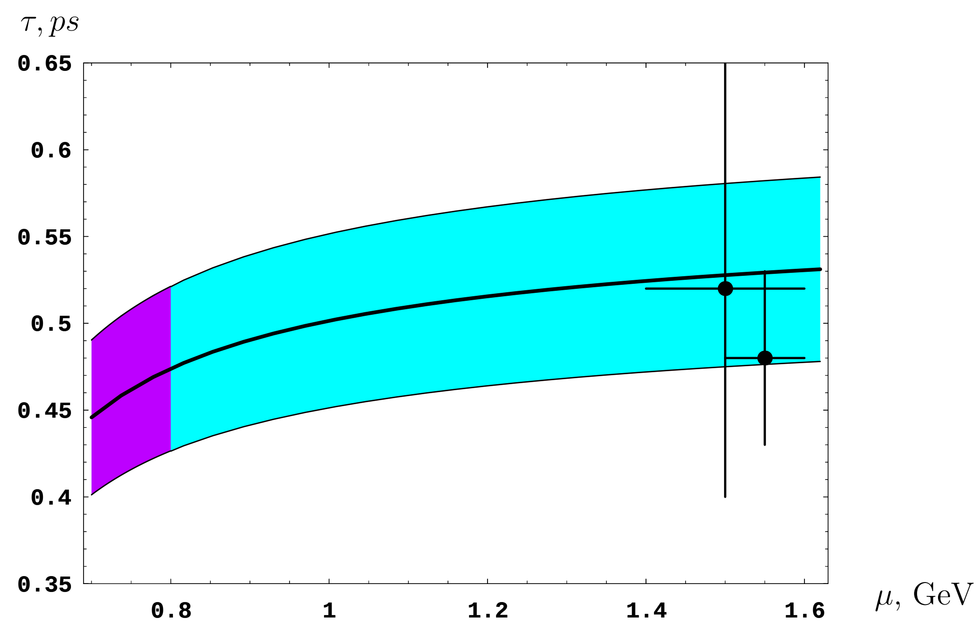

Figure 14: The dependence of B_c meson lifetime on the scale μ in the effective Hamiltonian (37). The shaded region shows the uncertainty of estimates, the dark shaded region is the preferable choice as given by the lifetimes of charmed mesons. The dots represent the values in the OPE approach.

The width of beauty decay in the sum rules was derived using the similar methods in [13]: $\Gamma(B_c^+ \to \bar{c}c + X) = (28 \pm 5) \cdot 10^{-14}$ GeV. The width of the electroweak annihilation is taken from [9] as

$12 \cdot 10^{-14}$ GeV.

In Fig. 14 we present the B_c meson lifetime calculated in the QCD SR under consideration. We also show the results of the lifetime evaluation in the framework of Operator Product Expansion in NRQCD [6, 7].

In contrast to OPE, where the basic uncertainty is given by the variation of heavy quark masses, these parameters are fixed by the two-point sum rules for bottomonia and charmonia, so that the accuracy of SR calculations for the total width of B_c is determined by the choice of scale μ for the hadronic weak lagrangian in decays of charmed quark. We show this dependence in Fig. 14, where $\frac{m_c}{2} < \mu < m_c$ and the dark shaded region corresponds to the scales preferred by data on the charmed meson lifetimes. The discussion on the optimal choice of scale in hadronic decays is addressed in the next section.

6 Discussion on the lifetimes of heavy hadrons

At present the ordinary prescription for the normalization point of lagrangian generating the non-leptonic decays of heavy quark Q, is $\mu \simeq m_Q$. The motivation is the following: the characteristic scale is determined by the energy release given by the heavy quark mass. Therefore, we can argue the operator product expansion in the inverse powers of m_Q, wherein we can factorize the Wilson coefficients taken in the perturbative QCD and the matrix elements of operators over the hadronic states with μ usually posed to m_Q. This prescription is in a qualitative agreement with the current data on the measured lifetimes of charmed and beauty hadrons and their branching ratios for the semileptonic decay modes, say.

Let us consider this issue in more details. To the moment, the analysis of decays in the QCD sum rules is restricted by the leading order (LO) in α_s (except the Coulomb-like corrections in the heavy quarkonia). The corresponding parameters, the heavy quark masses, are also fixed to the same order (they have to be reevaluated in next-to-leading order). Therefore, for the sake of consistency, we use the LO expressions, which are given by the partonic approximation improved by $1/m_Q$-corrections in HQET or NRQCD. In this way, we can write down the following formulae:

1. The semileptonic branching fraction of D^0 meson is given by

$$\mathrm{BR}_{sl}[D^0] = \frac{1}{2 + 2C_+^2(\mu) + C_-^2(\mu)}. \tag{40}$$

2. The difference between the total widths of charmed mesons is determined by the Pauli interference in decays of D^+, so that [36]

$$\Gamma[D^+] - \Gamma[D^0] = \cos^4\theta_c G_\mathrm{F}^2 \frac{m_c^3 f_D^2}{8\pi} \left[(C_+^2(\mu) - C_-^2(\mu))B + \frac{1}{3}(C_+^2(\mu) + C_-^2(\mu))\tilde{B} \right], \tag{41}$$

where θ_c is the Cabibbo angle, f_D is the leptonic constant of D meson, and the 'bag' constants are defined by

$$\langle D|(\bar{c}\Gamma_\mu q)(\bar{q}\Gamma_\mu c)|D\rangle = f_D^2 M_D^2 B, \tag{42}$$

$$\langle D|(\bar{c}\Gamma_\mu c)(\bar{q}\Gamma_\mu q)|D\rangle = \frac{1}{3} f_D^2 M_D^2 \tilde{B}, \tag{43}$$

with $\Gamma_\mu = \gamma_\mu(1-\gamma_5)$.

The similar expression can be derived for the beauty mesons [36]

$$\Gamma[B^+]-\Gamma[B^0]=|V_{bc}|^2G_{\rm F}^2\frac{m_b^3f_B^2}{8\pi}\left[(C_+^2(\mu)-C_-^2(\mu))B+\frac{1}{3}(C_+^2(\mu)+C_-^2(\mu))\tilde B\right]. \tag{44}$$

3. The semileptonic branching fraction of B^0 meson is given by

$$\mathrm{BR}_{sl}[B^0]=\frac{1}{2+0.22+[2C_+^2(\mu)+C_-^2(\mu)](1+k)}, \tag{45}$$

where the fraction of 0.22 is due to the $\tau\nu_\tau$-contribution, while the value of k denotes the fraction of $b\to c\bar c s$ transition in the nonleptonic decays. In the same way, the average yield of charm in the decays of beauty mesons is equal to

$$n_c=\frac{2+0.22+[2C_+^2(\mu)+C_-^2(\mu)](1+2k)}{2+0.22+[2C_+^2(\mu)+C_-^2(\mu)](1+k)}. \tag{46}$$

As for numerical applications, one usually puts
$\mu_D=m_c$ in decays of D mesons,
$\mu_B=m_b$ in decays of B mesons,
$f_D\approx f_B\approx 200$ MeV, and
$B=\tilde B=1$ naively motivated by nonrelativistic potential models.

At $k=0.4$ [37], this set (marked as SETMQ column) results in the estimates shown in Table 4 in comparison with the experimental data [33].

quantity	exp.	SETMQ	SETMQ$_2$	SETH
$\mathrm{BR}_{sl}[D^0]$, %	8.1 ± 1.1	15.4	15.4	8.6
$\Gamma[D^+]-\Gamma[D^0]$, ps^{-1}	-1.56 ± 0.03	-1.26	-0.19	-1.53
$\Gamma[D^0]/\Gamma[D_s]$	1.12 ± 0.05	1.00	1.00	1.11
$\mathrm{BR}_{sl}[B^0]$, %	10.45 ± 0.21	14	14	10.2
$\Gamma[B^+]-\Gamma[B^0]$, ps^{-1}	-0.043 ± 0.017	-0.022	0.024	-0.044
$\Gamma[B^0]/\Gamma[B_s]$	1.00 ± 0.05	1.00	1.00	1.05
n_c	1.12 ± 0.05	1.20	1.20	1.12
$\Gamma[B^0]/\Gamma[\Lambda_b]$	0.81 ± 0.05	1.00	1.00	0.81

Table 4: The comparison of theoretical estimates at various sets of parameters with the experimental data on the decays of heavy mesons.

First, we note that the semileptonic width of D^0 is well described to the given order and at chosen value of m_c, while its branching ratio is in a valuable contradiction with the data indicating a more higher enforcement of nonleptonic modes. Second, the qualitative agreement of predictions with the measured differences of $\Gamma[D^+]-\Gamma[D^0]$ and $\Gamma[B^+]-\Gamma[B^0]$ is mainly based on the assumption of $\tilde B\approx 1$. Recent consideration of charmed baryon lifetimes by M.Voloshin [38] clearly drawn a conclusion that the naive picture of color structure as given by the potential models (i.e. the purely antisymmetric color-composition of valence flavors) is significantly broken. A similar statement was obtained in the

description of D^* meson production at HERA, where the authors of [39] found that the $O(\alpha_{em}\alpha_s^3)$-calculations for the differential cross sections of $c\bar{q}$-pair composing the meson, are able to reproduce the measured spectra, if we introduce the valuable contribution by the color-octet state in addition to the singlet one. So, the four-quark singlet-operator results in

$$O_{(1)} = \langle D^*|(\bar{c}\gamma_\mu q)(\bar{q}\gamma_\mu c)|D^*\rangle \cdot \left(-g_{\mu\nu} + \frac{p_\mu p_\nu}{p^2}\right)\frac{1}{12M}, \tag{47}$$

which is reduced to $O_{(1)} = |\Psi_{(1)}(0)|^2$ in the framework of nonrelativistic potential model, where

$$|D^*\rangle = \sqrt{2M}\int \frac{d^3q}{(2\pi)^3}\Psi_{(1)}(q)\frac{\delta_{ij}}{\sqrt{3}} \cdot \frac{1}{\sqrt{2}}\bar{c}_i\not{\epsilon} q_j|0\rangle, \tag{48}$$

with ϵ_α denoting the polarization vector and $\Psi_{(1)}(q)$ being the wave function.

The term of color-octet was parameterized by

$$O_{(8)} = \langle D^*|(\bar{c}\lambda^a\gamma_\mu q)(\bar{q}\lambda^a\gamma_\mu c)|D^*\rangle \cdot \left(-g_{\mu\nu} + \frac{p_\mu p_\nu}{p^2}\right)\frac{1}{64M}, \tag{49}$$

which is, in a similar way, can be represented as $O_{(8)} = |\Psi_{(8)}(0)|^2$, if we introduce the additional Fock state

$$\sqrt{2M}\int \frac{d^3q}{(2\pi)^3}\Psi_{(8)}(q)\frac{\lambda^a_{ij}}{\sqrt{2}}n_a \cdot \frac{1}{\sqrt{2}}\bar{c}_i\not{\epsilon} q_j|0\rangle, \tag{50}$$

where the $\bar{c}$ and q fields represent the rapid valence quarks, and n_a is a random color-vector determined by soft degrees of freedom inside the meson (i.e. by the quark-gluon sea). We have $\langle n_a n_b\rangle = \delta_{ab}$ in the production, while $\langle n_a n_b\rangle = \frac{1}{8}\delta_{ab}$ in decays.

The data on the D^* production in DIS give $O_{(8)}/O_{(1)} \simeq 1.3$. We can analogously expect that in the D meson the ratio of four-quark matrix elements is close to that in D^*. So, $O_{(8)}[D]/O_{(1)}[D] \simeq 1$, where $O_{(1,8)}[D]$ can be obtained from the above expressions for D^* by the substitution of $\gamma_5\gamma_\mu$ for γ_μ and removing the transverse projector. Then we straightforwardly find

$$\tilde{B} = 1 + \frac{O_{(8)}[D]}{O_{(1)}[D]} \approx 2. \tag{51}$$

Putting $\tilde{B} = 2$ into the SETMQ we get the values given in the SETMQ$_2$ column in Table 4. So, the choice of $\mu = m_Q$ is in a deep contradiction with the observed differences of total widths for the heavy mesons at the most reasonable value of $\tilde{B} = 2$.

In this position we argue the following: There are other physical scales in the problem, which are characteristic for two hadronic systems in the decay process. The first system is the decaying hadron. The second is the transition current $c \to s$ or $\bar{b} \to \bar{c}$, where the form factor behaviour versus the transferred momentum is determined by the $c\bar{s}$ and $\bar{b}c$ states. Those hadronic systems have the following scales, being the average squares of heavy quark momentum $\langle p^2\rangle$, which are phenomenologically equal to

$$\mu^2_{c\bar{u}} = \mu^2_{b\bar{d}} = 2T\bar{\Lambda}, \tag{52}$$

where according to the potential models $\bar{\Lambda} \simeq 0.4$ GeV is the binding energy of heavy quark (i.e. the constituent mass of light quark), $T \simeq 0.45$ GeV is the average kinetic energy in the system. T

determines the 2S-1S splitting and it is approximately independent of quark flavors. Analogously, we put

$$\mu^2_{c\bar{s}} = \mu^2_{b\bar{s}} = 2T\bar{\Lambda}_s, \tag{53}$$

where $\bar{\Lambda}_s = \bar{\Lambda} + (m_{D_s} - m_D) = \bar{\Lambda} + (m_{B_s} - m_B) \simeq 0.5$ GeV. For the $b \to c$ current we put

$$\mu^2_{c\bar{b}} = 2T m_{bc}, \tag{54}$$

with $m_{bc} \simeq m_b m_c/(m_b + m_c) \simeq m_B m_D/(m_B + m_D) \approx 1.3$ GeV.

Let us suppose that the decay scale is given by the following combinations:

$$\begin{aligned} \mu^2_D &= \mu_{c\bar{u}} \cdot \mu_{c\bar{s}}, \\ \mu^2_{D_s} &= \mu^2_{c\bar{s}}, \\ \mu^2_B &= \mu_{b\bar{u}} \cdot \mu_{c\bar{b}}, \\ \mu^2_{B_s} &= \mu_{b\bar{s}} \cdot \mu_{c\bar{b}}, \end{aligned} \tag{55}$$

At $\tilde{B} = 2$, this set of parameters with $f_B \cong f_D \cong 175$ MeV, and $k = 0.18$ is represented by the SETH column in Table 4. We see a good agreement with the data. The result of μ_B and k variations is also shown in Fig. 15.

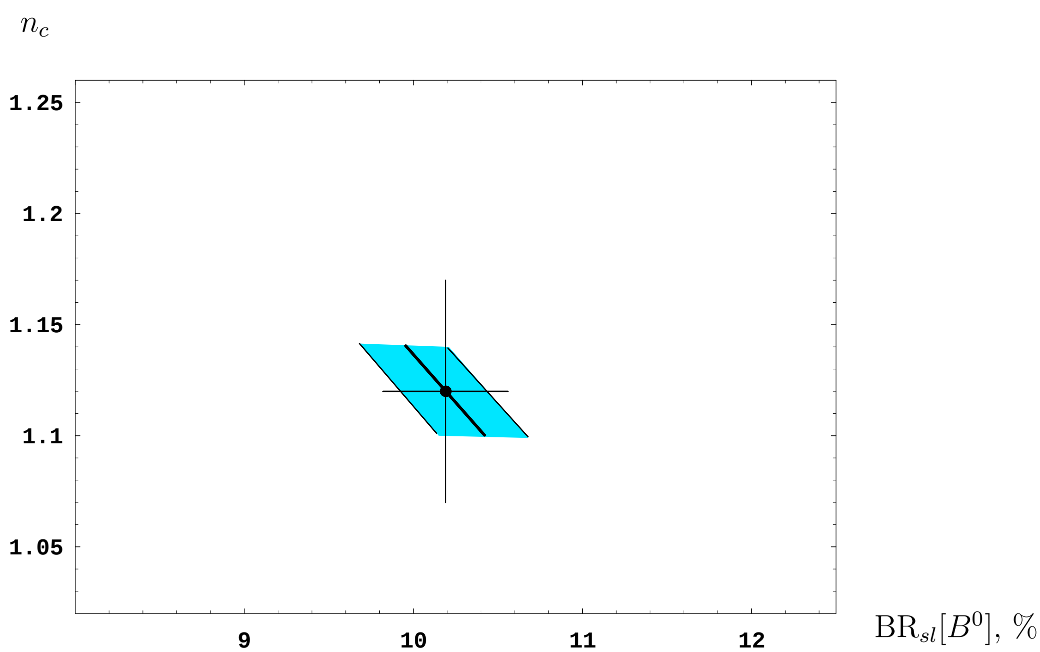

Figure 15: The predictions on the semileptonic branching ratio of B^0 meson and the average yield of charmed quarks in its decays with the $\pm 3\%$-variation of hadronic scale in the nonleptonic effective lagrangian and the change of doubly charmed mode fraction $k = 0.15 \div 0.22$ in comparison with the ARGUS and CLEO data shown by the dot with the error bars.

We stress that the theoretical OPE prediction for the contribution of $b \to c\bar{c}s$ mode, $k = 0.4$, is significantly overestimated, to our opinion. Indeed, looking at the semileptonic decay with $\tau^+\nu_\tau$ in the final state we see that at the same values of form factors as in modes with light leptons the

heavy lepton mode is suppressed because of the restricted phase space, so that the reduction factor equals 0.22. The same effect has to take place in the b-quark decays with two charmed quarks in the final state. So, since the sum of D and K masses is greater than the τ mass we could expect even the greater suppression than for the heavy lepton mode. Nevertheless, OPE operates with the quark masses at so moderate release of energy, that certainly leads to the overestimation of phase space in the decay of $b \to c\bar{c}s$. We use more realistic values for k close to 0.2.

Next, the contribution of four-quark operators are suppressed in decays of Λ_b baryon. So, one expects that the deviation of its total width from that of B mesons is about 2-3% [40]. To the moment, the experimental result is far away from this expectation (see Table 4). We can reproduce the data on the Λ_b lifetime, if we introduce the light diquark scale

$$\mu^2_{ud} = 2\frac{T}{2}\bar{\Lambda}, \tag{56}$$

where $T/2$ corresponds to the half tension of color string inside the diquark. Then we pose

$$\mu^2_{\Lambda_b} = \mu_{ud} \cdot \mu_{c\bar{b}}, \tag{57}$$

which result in a good agreement with the experimental data.

To finish this discussion we draw the conclusion: a probable way to reach the agreement between the theoretical predictions and available experimental data on the lifetimes and inclusive decay widths of heavy hadrons is to suggest the different normalization points in the effective nonleptonic lagrangian for the heavy quark weak decays as dependent on the hadron. This assumption provide us with quite acceptable results to the leading order in α_s. The variation of normalization point shows the sensitivity of calculations to the higher orders in the QCD coupling constant, which indicates, first, the necessity to proceed with the higher orders, and, second, the appropriate choice of scale can allow us to decrease the scale-dependent higher orders terms.

Thus, we suppose that the preferable choice of scale in the $c \to s$ decays of B_c is equal to

$$\mu^2_{B_c} = \mu_{c\bar{b}} \cdot \mu_{c\bar{s}} \approx (0.85 \text{ GeV})^2, \tag{58}$$

and at $a_1(\mu_{B_c}) = 1.20$ in the charmed quark decays we predict

$$\tau[B_c] = 0.48 \pm 0.05 \text{ ps}. \tag{59}$$

7 Conclusion

We have investigated the semileptonic decays of B_c meson due to the weak decays of charmed quark in the framework of three-point sum rules in QCD. We have found the important role played by the Coulomb-like α_s/v-corrections. As in the case of two-point sum rules, the form factors are about three times enhanced due to the Coulomb renormalization of quark-meson vertex for the heavy quarkonium B_c. We have studied the dependence of form factors on the threshold energy, which determines the continuum region of $\bar{b}s$ system. The obtained dependence has the stability region, serving as the test of convergency for the sum rule method. The HQET two-point sum rules for the leptonic constant f_{B_s} and $f_{B^*_s}$ have been reanalyzed to introduce the term caused by the product of quark and gluon condensates. This contribution essentially improves the stability of SR results for the leptonic constants of B mesons, yielding: $f_B = 140 \div 170$ MeV.

We have studied the soft limit for the form factors in combined HQET/NRQCD technique at the recoil momentum close to zero, which allows us to derive the generalized relations due to the spin symmetry of effective lagrangian. The relations are in a good agreement with the full QCD results, which means that the corrections to the form factors in both relative velocity of heavy quarks inside the $\bar{b}c$ quarkonium and the inverse heavy quark masses are small within the accuracy of the method.

Next, we have studied the nonleptonic decays, using the assumption on the factorization of the weak transition. The results on the widths and branching fractions for various decay modes of B_c are collected in Tables.

Finally, we have estimated the B_c meson lifetime, and showed the dependence on the scale for the hadronic weak lagrangian in decays of charmed quark

$$\tau[B_c] = 0.48 \pm 0.05 \text{ ps}.$$

Our estimates are in a good agreement with the theoretical predictions for the lifetime in both the potential models and OPE as well as with the experimental data.

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8 Appendix A

For the perturbative spectral densities $\rho_i(s_1, s_2, Q^2)$ we have the following expressions [13]:

$$\begin{aligned}
\rho_+(s_1, s_2, Q^2) &= \frac{3}{2k^{3/2}}\{\frac{k}{2}(\Delta_1+\Delta_2) - k[m_3(m_3-m_1)+m_3(m_3-m_2)] - \\
&\quad [2(s_2\Delta_1+s_1\Delta_2)-u(\Delta_1+\Delta_2)] \\
&\quad \cdot[m_3^2-\frac{u}{2}+m_1m_2-m_2m_3-m_1m_3]\},
\end{aligned} \tag{60}$$

$$\begin{aligned}
\rho_-(s_1, s_2, Q^2) &= -\frac{3}{2k^{3/2}}\{\frac{k}{2}(\Delta_1-\Delta_2) - k[m_3(m_3-m_1)-m_3(m_3-m_2)] + \\
&\quad [2(s_2\Delta_1-s_1\Delta_2)+u(\Delta_1-\Delta_2)] \\
&\quad \cdot[m_3^2-\frac{u}{2}+m_1m_2-m_2m_3-m_1m_3]\},
\end{aligned} \tag{61}$$

$$\begin{aligned}
\rho_V(s_1, s_2, Q^2) &= \frac{3}{k^{3/2}}\{(2s_1\Delta_2-u\Delta_1)(m_3-m_2) \\
&\quad +(2s_2\Delta_1-u\Delta_2)(m_3-m_1)+m_3k\},
\end{aligned} \tag{62}$$

$$\begin{aligned}
\rho_0^A(s_1, s_2, Q^2) &= \frac{3}{k^{1/2}}\{(m_1-m_3)[m_3^2+\frac{1}{k}(s_1\Delta_2^2+s_2\Delta_1^2-u\Delta_1\Delta_2)] \\
&\quad -m_2(m_3^2-\frac{\Delta_1}{2})-m_1(m_3^2-\frac{\Delta_2}{2}) \\
&\quad +m_3[m_3^2-\frac{1}{2}(\Delta_1+\Delta_2-u)+m_1m_2]\},
\end{aligned} \tag{63}$$

$$\begin{aligned}
\rho_+^A(s_1, s_2, Q^2) &= \frac{3}{2k^{3/2}}\{m_1[2s_2\Delta_1-u\Delta_2+4\Delta_1\Delta_2+2\Delta_2^2] \\
&\quad m_1m_3^2[4s_2-2u]+m_2[2s_1\Delta_2-u\Delta_1]-m_3[2(3s_2\Delta_1+s_1\Delta_2) \\
&\quad -u(3\Delta_2+\Delta_1)+k+4\Delta_1\Delta_2+2\Delta_2^2+m_3^2(4s_2-2u)] \\
&\quad +\frac{6}{k}(m_1-m_3)[4s_1s_2\Delta_1\Delta_2-u(2s_2\Delta_1\Delta_2+s_1\Delta_2^2+s_2\Delta_1^2)
\end{aligned} \tag{64}$$

$$
\begin{aligned}
& +2s_2(s_1\Delta_2^2+s_2\Delta_1^2)]\}, \\
\rho_-^A(s_1,s_2,Q^2) \;\; = \;\; & -\frac{3}{2k^{5/2}}\{kum_3(2m_1m_3-2m_3^2+u)+12(m_1-m_3)s_2^2\Delta_1^2+ \\
& k\Delta_2[(m_1+m_3)u-2s_1(m_2-m_3)]+2\Delta_2^2(k+3us_1)(m_1-m_3) \\
& +\Delta_1[ku(m_2-m_3)+2\Delta_2(k-3u^2)(m_1-m_3)]+ \\
& 2s_2(m_1-m_3)[2km_3^2-k\Delta_1+3u\Delta_1^2-6u\Delta_1\Delta_2]- \\
& 2s_1s_2(km_3-3\Delta_2^2(m_1-m_3))],
\end{aligned} \tag{65}
$$

$$
\begin{aligned}
\rho_+^{\prime A}(s_1,s_2,Q^2) \;\; = \;\; & -\frac{3}{2k^{5/2}}\{-2(m_1-m_3)[(k-3us_2)\Delta_1^2+6s_1^2\Delta_2^2]+ \\
& ku(m_1-m_3)(2m_3^2+\Delta_2)+ku^2m_3+\Delta_1[ku(2m_1-m_2-3m_3) \\
& -2(m_1-m_3)(ks_2-k\Delta_2+3u^2\Delta_2)]- \\
& 2s_1[(m_1-m_3)(2km_3^2-6u\Delta_1\Delta_2-3u\Delta_2^2)+ \\
& 2s_2(km_3+3m_1\Delta_1^2-3m_3\Delta_1^2)+k\Delta_2(2m_1-m_2-3m_3)]\},
\end{aligned} \tag{66}
$$

$$
\begin{aligned}
\rho_-^{\prime A}(s_1,s_2,Q^2) \;\; = \;\; & \frac{3}{2k^{5/2}}\{2(m_1-m_3)[(k+3us_2)\Delta_1^2+6s_1^2\Delta_2^2]+ \\
& ku(m_1-m_3)(2m_3^2+\Delta_2)+ku^2m_3+\Delta_1[ku(-2m_1-m_2+m_3) \\
& -2(m_1-m_3)(ks_2-k\Delta_2+3u^2\Delta_2)]+ \\
& 2s_1[(m_1-m_3)(2km_3^2-6u\Delta_1\Delta_2+3u\Delta_2^2)- \\
& 2s_2(km_3-3m_1\Delta_1^2+3m_3\Delta_1^2)+k\Delta_2(2m_1+m_2-m_3)]\}.
\end{aligned} \tag{67}
$$

Here $k=(s_1+s_2+Q^2)^2-4s_1s_2$, $u=s_1+s_2+Q^2$, $\Delta_1=s_1-m_1^2+m_3^2$ and $\Delta_2=s_2-m_2^2+m_3^2$. m_1, m_2 and m_3 are the masses of quark flavours relevant to the various decays, see prescriptions in Fig. 1.

9 Appendix B

Here we list the expression for the form factors of semileptonic decays $B_c \to B_s^{(*)}$ taken from the potential model [32].

$$
f_+ \;\; = \;\; \frac{(\tilde{m}_c+\tilde{m}_s)}{2\tilde{m}_s}\sqrt{\frac{M_{B_s}}{M_{B_c}}}\xi(w), \tag{68}
$$

$$
f_- \;\; = - \;\; \frac{(\tilde{m}_c-\tilde{m}_s+2\tilde{m}_b)}{2\tilde{m}_s}\sqrt{\frac{M_{B_s}}{M_{B_c}}}\xi(w), \tag{69}
$$

$$
F_0^A \;\; = \;\; \frac{M_{B_c}^2+M_{B_s^*}^2-q^2+2M_{B_c}(\tilde{m}_s-\tilde{m}_b)}{2\tilde{m}_s}\sqrt{\frac{M_{B_s^*}}{M_{B_c}}}\xi(w), \tag{70}
$$

$$
F_+^A \;\; = \;\; -\frac{1-2\tilde{m}_b/M_{B_c}}{2\tilde{m}_s}\sqrt{\frac{M_{B_s^*}}{M_{B_c}}}\xi(w), \tag{71}
$$

$$
F_-^A \;\; = \;\; \frac{1+2\tilde{m}_b/M_{B_c}}{2\tilde{m}_s}\sqrt{\frac{M_{B_s^*}}{M_{B_c}}}\xi(w), \tag{72}
$$

where

$$\xi(w) = \frac{2\omega\omega_x}{\omega^2+\omega_x^2}\sqrt{\frac{2\omega\omega_x}{\omega^2 w^2+\omega_x^2}}\exp\left(-\frac{\tilde{m}_b^2(w^2-1)}{\omega^2 w^2+\omega_x^2}\right), \tag{73}$$

$$\omega = 2\pi\left(\frac{M_{B_c}\tilde{f}_{B_c}^2}{12}\right)^{1/3}, \quad \omega_x = 2\pi\left(\frac{M_{B_s^{(*)}}\tilde{f}_{B_s^{(*)}}^2}{12}\right)^{1/3}, \tag{74}$$

where w is the product of B_c and $B_s^{(*)}$ four-velocities. The quark masses and the leptonic constants have the values generally used in the calculations in the framework of potential models

$$\tilde{m}_b = 4.8 \text{ GeV}, \quad \tilde{m}_c = 1.5 \text{ GeV}, \quad \tilde{m}_s = 0.55 \text{ GeV}, \quad \tilde{f}_{B_c} = 0.47 \text{ GeV}, \quad \tilde{f}_{B_s^{(*)}} = 0.17 \text{ GeV}.$$

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